MAVZU. Chiziqli operatorlar fazosi. Chiziqli operatorlarning o'tish matritsasi.

Matritsalar algebrasining asosiy tushunchalaridan biri – chiziqli operatorlar tushunchasidir. Faraz qilaylik bizga $L,\ L_1$ chiziqli fazolar berilgan bo'lsin.

1- ta'rif. Agar biror A qoida yoki qonun bo'yicha har bir $x \in L$ elementga $y \in L_1$ element mos qo'yilgan bo'lsa, u holda L fazoni L_1 fazoga o'tkazuvchi A *operator (almashtirish, akslantirish)* aniqlangan deyiladi va $y = A(x)$ ko'rinishda belgilanadi.

2- ta'rif. Agar ixtiyoriy $x, y \in L,\ \lambda \in R$ uchun:

1) $\tilde{A}(x+y) = \tilde{A}(x) + \tilde{A}(y)$ (*operatorning additivligi*);

2) $\tilde{A}(\lambda x) = \lambda \tilde{A}(x)$ (*operatorning bir jinsliligi)* munosabatlar o'rinli bo'lsa, u holda bu operator chiziqli operator deyiladi.

1- misol. $\tilde{A}: R^2 \to R^3$ operator $\tilde{A}(x,y) = (x, y, x+y)$ qoida bilan aniqlangan bo'lsin, u holda bu operatorning chiziqli operator ekanligini ko'rsating.

Yechish. Ma'lumki, $\vec{a}_1 = (x_1, y_1)$ va $\vec{a}_2 = (x_2, y_2)$ vektor uchun $\vec{a}_1 + \vec{a}_2 = (x_1 + x_2, y_1 + y_2)$. U holda $\vec{a}_1 + \vec{a}_2 = (x_1 + x_2, y_1 + y_2)$ elementga A operatorni ta'sir ettirsak, quyidagiga ega bo'lamiz:

$$\tilde{A}(a_1 + a_2) = \tilde{A}(x_1 + x_2, y_1 + y_2) = (x_1 + x_2, y_1 + y_2, x_1 + x_2 + y_1 + y_2) =$$
$$= (x_1, y_1, x_1 + y_1) + (x_2, y_2, x_2 + y_2) = \tilde{A}(a_1) + \tilde{A}(a_2).$$

Bu esa $\tilde{A}$ operatorning additivligini ko'rsatadi.

Endi operatorning bir jinsli ekanligini tekshiramiz. Ma'lumki, $ka_1 = (kx_1, ky_1)$. U holda

$$\tilde{A}(ka_1) = \tilde{A}(kx_1, ky_1) = (kx_1, ky_1, kx_1 + ky_1) = k(x_1, y_1, x_1 + y_1) = k\tilde{A}(a_1).$$

Demak, biz o'rganayotgan operator chiziqli operatordir.

$y = A(x) \in L_1$ element $x \in L$ elementning *aksi,* $x \in L$ elementning o'zi esa $y \in L_1$ elementning *proobrazi* deyiladi. Agar $L = L_1$ bo'lsa, u holda $\tilde{A}$ operator L fazoni o'zini o'ziga akslantiruvchi operator bo'ladi. Biz ko'proq fazoni o'zini o'ziga akslantiruvchi operatorlarni o'rganamiz.

1-teorema. Har bir $\tilde{A}: L^n \to L^n$ chiziqli operatorga berilgan bazisda $n-$ tartibli matritsa mos keladi va aksincha har bir $n-$tartibli matritsaga n o'lchovli chiziqli fazoni, n o'lchovli chiziqli fazoga akslantiruvchi $\tilde{A}$ chiziqli operator mos keladi.

Isbot. Faraz qilaylik $\tilde{A}: L^n \to L^n$ chiziqli operator bo‘lsin. Agar $\left\{\vec{e}_1, \vec{e}_2, ..., \vec{e}_n\right\} \subseteq L^n$ vektorlar sistemasi L^n fazoning bazisi bo‘lsa, u holda ixtiyoriy $\vec{x} \in L^n$ elementni bu bazis elementlari orqali yozish mumkin:

$$\vec{x} = x_1\vec{e}_1 + ... + x_n\vec{e}_n. \qquad (1)$$

Bu yerda biz $\tilde{A}$ operatorning chiziqliligidan foydalanib, $\tilde{A}(x)$ni quyidagicha yoza olamiz:

$$\tilde{A}(x) = \tilde{A}\left(x_1\vec{e}_1 + ... + x_n\vec{e}_n\right) = x_1\tilde{A}(\vec{e}_1) + ... + x_n\tilde{A}(\vec{e}_n). \qquad (2)$$

Bu yerda har bir $\tilde{A}(e_i)$ $(i = \overline{1,n})$ elementlar o‘z navbatida L^n fazoning elementlari bo‘lganligi sababli, bu elementlarni ham $\left\{\vec{e}_1, \vec{e}_2, ..., \vec{e}_n\right\}$ bazis orqali yozish mumkin:

$$\tilde{A}(\vec{e}_i) = a_{1i}\vec{e}_1 + ... + a_{ni}\vec{e}_n. \qquad (3)$$

U holda (3) dan foydalanib (2) ifodani quyidagicha yozish mumkin:

$$\begin{aligned}\tilde{A}(x) &= x_1(a_{11}\vec{e}_1 + a_{21}\vec{e}_2 + ... + a_{n1}\vec{e}_n) + x_2(a_{12}\vec{e}_1 + a_{22}\vec{e}_2 + ... + a_{n2}\vec{e}_n) + ...\\ &+ x_n(a_{1n}\vec{e}_1 + a_{2n}\vec{e}_2 + ... + a_{nn}\vec{e}_n) = (a_{11}x_1 + a_{12}x_2 + ... + a_{1n}x_n)\vec{e}_1 +\\ &+ (a_{21}x_1 + a_{22}x_2 + ... + a_{2n}x_n)\vec{e}_2 + ... + (a_{n1}x_1 + a_{n2}x_2 + ... + a_{nn}x_n)\vec{e}_n\end{aligned} \qquad (4)$$

Ikkinchi tomondan $y = \tilde{A}(x)$ element ham $\left\{\vec{e}_1, \vec{e}_2, ..., \vec{e}_n\right\}$ bazis elementlari bo‘yicha quyidagi yoyilmaga ega:

$$y = \tilde{A}(x) = y_1\vec{e}_1 + y_2\vec{e}_2 + ... + y_n\vec{e}_n. \qquad (5)$$

Vektorning bitta bazis bo‘yicha yoyilmasi yagonaligidan (4) va (5) tengliklarning o‘ng tomonlarini tenglashtirib, quyidagini olamiz.

$$\begin{cases} y_1 = a_{11}x_1 + a_{12}x_2 + ... + a_{1n}x_n \\ y_2 = a_{21}x_1 + a_{22}x_2 + ... + a_{2n}x_n \\ .... \\ y_n = a_{n1}x_1 + a_{n2}x_2 + ... + a_{nn}x_n \end{cases}$$

yoki matritsa ko‘rinishida $Y = AX$, bu yerda

$$A = \begin{pmatrix} a_{11} & a_{12} & ... & a_{1n} \\ a_{21} & a_{22} & ... & a_{2n} \\ . & . & ... & . \\ a_{n1} & a_{n2} & ... & a_{nn} \end{pmatrix}, X = \begin{pmatrix} x_1 \\ x_2 \\ . \\ x_n \end{pmatrix}, \ Y = \begin{pmatrix} y_1 \\ y_2 \\ . \\ y_n \end{pmatrix}$$

3- ta’rif. $A = (a_{ij})$ $(i, j = 1, 2, ..., n)$ matritsa $\tilde{A}$ operatorning $\left\{\vec{e}_1, \vec{e}_2, ..., \vec{e}_n\right\}$ bazisdagi matritsasi, $A = (a_{ij})$ $(i, j = 1, 2, ..., n)$ matritsaning rangi esa $\tilde{A}$ *operatorning rangi* deyiladi.

L fazoning barcha vektorlarini θ nol vektorga akslantiruvchi $\theta(x)=\theta$ operator *nol operator*, $E(x)=x$ tenglikni qanoatlantiruvchi operator *birlik operator* deb ataladi.

2- misol. R^3 fazoda $\{e_1,e_2,e_3\}$ bazisda chiziqli operator matritsasi

$$A=\begin{pmatrix} 3 & 2 & 4 \\ -1 & 5 & 6 \\ 1 & 8 & 2 \end{pmatrix}$$

berilgan bo'lsin. $x=4\vec{e}_1-3\vec{e}_2+\vec{e}_3$ vektorning $y=A(x)$ aksini toping.

Yechish. Yuqorida qayd qilingan formulaga ko'ra

$$\begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix}=\begin{pmatrix} 3 & 2 & 4 \\ -1 & 5 & 6 \\ 1 & 8 & 2 \end{pmatrix}\begin{pmatrix} 4 \\ -3 \\ 1 \end{pmatrix}=\begin{pmatrix} 10 \\ -13 \\ -18 \end{pmatrix}$$

Demak, $y=10\vec{e}_1-13\vec{e}_2-18\vec{e}_3$.

3- misol. $T:R^3\to R^4;\ T\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}=\begin{pmatrix} x_1+x_2 \\ x_1-x_2 \\ x_3 \\ x_1 \end{pmatrix}$ operatorning matritsasini toping.

Yechish. $A=\left[T(\vec{e}_1)\quad T(\vec{e}_2)\quad T(\vec{e}_3)\right]$ matritsaning har bir elementini topamiz:

$$T(\vec{e}_1)=T\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}=\begin{pmatrix} 1+0 \\ 1-0 \\ 0 \\ 1 \end{pmatrix}=\begin{pmatrix} 1 \\ 1 \\ 0 \\ 1 \end{pmatrix},$$

$$T(\vec{e}_2)=T\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}=\begin{pmatrix} 0+1 \\ 0-1 \\ 0 \\ 0 \end{pmatrix}=\begin{pmatrix} 1 \\ -1 \\ 0 \\ 0 \end{pmatrix},$$

$$T(\vec{e}_3)=T\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}=\begin{pmatrix} 0+0 \\ 0-0 \\ 1 \\ 0 \end{pmatrix}=\begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}.$$

U holda

$$A=\begin{pmatrix}1 & 1 & 0\\ 1 & -1 & 0\\ 0 & 0 & 1\\ 1 & 0 & 0\end{pmatrix}.$$

Chiziqli operatorlar ustida bajariladigan chiziqli amallar bilan tanishib chiqamiz. L^n chiziqli fazoda $A,\ B$ operatorlar berilgan bo‘lsin.

4- ta’rif. $\left(A+B\right)(x)=A(x)+B(x)$ tenglik bilan aniqlanadigan operatorni $A,\ B$ *operatorlarning yig‘indisi* deb ataladi.

2-teorema. Agar A va B operatorlar chiziqli operatorlar bo‘lsa, u holda $A+B$ operator ham chiziqli operator bo‘ladi .

Isbot. Ixtiyoriy $x,y\in R^n$ vektorlar va $\alpha\in R$ son uchun:

1) $\left(\tilde{A}+\tilde{B}\right)(x+y)=\tilde{A}(x+y)+\tilde{B}(x+y)=\tilde{A}(x)+\tilde{A}(y)+\tilde{B}(x)+\tilde{B}(y)=$

$$=\left(\tilde{A}+\tilde{B}\right)(x)+\left(\tilde{A}+\tilde{B}\right)(y);$$

2) $\left(\tilde{A}+\tilde{B}\right)(\alpha x)=\tilde{A}(\alpha x)+\tilde{B}(\alpha x)=\alpha(\tilde{A}(x))+\alpha(\tilde{B}(x))=\alpha(\tilde{A}(x)+\tilde{B}(x))=$

$$=\alpha\left[(\tilde{A}+\tilde{B})(x)\right]$$

munosabatlar o‘rinli. Bu esa $A+B$ operator chiziqli ekanligini ko‘rsatadi.

5- ta’rif. $\left(\tilde{A}\tilde{B}\right)(x)=\tilde{B}\left(\tilde{A}(x)\right)$ tenglik bilan aniqlanadigan, ya‘ni $A,\ B$ operatorlarni ketma-ket bajarishdan hosil bo‘lgan $A\cdot B$ operator A *va* B *operatorlarning ko‘paytmasi* deyiladi.

3-teorema. Agar A va B operatorlar chiziqli operatorlar bo‘lsa, u holda $A\cdot B$ operator ham chiziqli operator bo‘ladi .

Isbot. Ixtiyoriy $\vec{x},\vec{y}\in R^n$ vektorlar va $\alpha\in R$ son uchun:

1) $\left(\tilde{A}\tilde{B}\right)(\vec{x}+\vec{y})=\tilde{B}[\tilde{A}(\vec{x}+\vec{y})]=\tilde{B}(\tilde{A}(\vec{x})+\tilde{A}(\vec{y}))=\left(\tilde{A}\tilde{B}\right)(\vec{x})+\left(\tilde{A}\tilde{B}\right)(\vec{y});$

2) $\left(\tilde{A}\tilde{B}\right)(\alpha\vec{x})=\tilde{B}[\tilde{A}(\alpha\vec{x})]=\tilde{B}[\alpha(\tilde{A}(\vec{x}))]=\alpha[\tilde{B}(\tilde{A}(\vec{x}))]=\alpha[(\tilde{A}\tilde{B})(\vec{x})]$

munosabat o‘rinli. Bu esa $A\cdot B$ operator chiziqli ekanligini ko‘rsatadi.

6- ta’rif. $\left(\alpha\tilde{A}\right)(\vec{x})=\alpha\left(\tilde{A}(\vec{x})\right)$ tenglik bilan aniqlanadigan αA operator A operatorlarning α *songa ko‘paytmasi* deyiladi.

4-teorema. Agar A operator chiziqli operator bo‘lsa, u holda αA operator ham chiziqli operator bo‘ladi .

Isbot. Ixtiyoriy,ixtiyoriy $\vec{x}, \vec{y} \in R^n$ vektorlar va $\alpha, \beta \in R$ sonlar uchun:

1) $\left(\alpha\tilde{A}\right)(\vec{x}+\vec{y}) = \alpha[\tilde{A}(\vec{x}+\vec{y})] = \alpha(\tilde{A}(\vec{x}) + \tilde{A}(\vec{y})) =$

$= \alpha(\tilde{A}(\vec{x})) + \alpha(\tilde{A}(\vec{y})) = \left(\alpha\tilde{A}\right)(\vec{x}) + \left(\alpha\tilde{A}\right)(\vec{y});$

2) $\left(\alpha\tilde{A}\right)(\beta\vec{x}) = \alpha[\tilde{A}(\beta\vec{x})] = \alpha[\beta(\tilde{A}(\vec{x}))] = \beta[\alpha(\tilde{A}(\vec{x}))] = \beta[(\alpha\tilde{A})(\vec{x})]$

munosabat o‘rinli. Bu esa αA operator chiziqli ekanligini ko‘rsatadi.

7- ta’rif. $\tilde{A}(x)$ operator uchun $\tilde{A}\tilde{A}^{-1} = \tilde{A}^{-1}\tilde{A} = \tilde{E}$ munosabat o‘rinli bo‘lsa, u holda A^{-1} operator A operatorga *teskari operator* deb ataladi.

5-teorema.

$\tilde{A}(x_1, x_2, x_3) = \left(2x_2, -2x_1 + 3x_2 + 2x_3, 4x_1 - x_2 + 5x_3\right)$ **va**

$\tilde{B}(x_1, x_2, x_3) = \left(-3x_1 + x_2, 2x_2 + x_3, -x_2 + 3x_3\right)$

operatorlar berilgan. $C = A \cdot B$ operator va uning matritsasi topilsin.

Yechish. Avval A va B matritsalarni topib olamiz:

$$\tilde{A}(\vec{e}_1) = \begin{pmatrix} 0 \\ -2 \\ 4 \end{pmatrix}, \ \tilde{A}(\vec{e}_2) = \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix}, \ \tilde{A}(\vec{e}_3) = \begin{pmatrix} 0 \\ 2 \\ 5 \end{pmatrix},$$

$$\tilde{B}(\vec{e}_1) = \begin{pmatrix} -3 \\ 0 \\ 0 \end{pmatrix}, \ B(\vec{e}_2) = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}, \ \tilde{B}(\vec{e}_3) = \begin{pmatrix} 0 \\ 1 \\ 3 \end{pmatrix}.$$

U holda

$$A = \begin{pmatrix} 0 & 2 & 0 \\ -2 & 3 & 2 \\ 4 & -1 & 5 \end{pmatrix}, \quad B = \begin{pmatrix} -3 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & -1 & 3 \end{pmatrix}.$$

$$C = AB = \begin{pmatrix} 0 & 4 & 2 \\ 6 & 2 & 9 \\ -12 & -3 & 14 \end{pmatrix}.$$

Bundan

$$\tilde{C}(\vec{e}_1) = \begin{pmatrix} 0 \\ 6 \\ -12 \end{pmatrix}, \tilde{C}(\vec{e}_2) = \begin{pmatrix} 4 \\ 2 \\ -3 \end{pmatrix}, \tilde{C}(\vec{e}_3) = \begin{pmatrix} 2 \\ 9 \\ 14 \end{pmatrix}.$$

$\tilde{C}(\vec{x}) = \left(4x_2 + 2x_3,\ 6x_1 + 2x_2 + 9x_3,\ -12x_1 - 3x_2 + 14x_3\right).$

Bitta chiziqli operatorning turli bazislardagi matritsalari orasidagi bog‘lanish haqidagi teoremani keltiramiz.

6-teorema. Agar A chiziqli operatorning $\left\{\vec{e}_1, \vec{e}_2, ..., \vec{e}_n\right\}$ va $\left\{\vec{e_1}^*, \vec{e_2}^*, ..., \vec{e_n}^*\right\}$ bazislardagi matritsalari mos ravishda A va A^* matritsalardan iborat bo‘lsa, u holda $A^* = C^{-1}AC$ munosabat o‘rinli bo‘ladi. Bu yerda C o‘tish matritsasi deb ataladi.

5- misol. $\{\vec{e}_1, \vec{e}_2\}$ bazisda chiziqli operator matritsasi $A = \begin{pmatrix} 17 & 6 \\ 6 & 8 \end{pmatrix}$ berilgan bo‘lsin. Yangi $\begin{cases} \vec{e_1}^* = \vec{e}_1 - 2\vec{e}_2 \\ \vec{e_2}^* = 2\vec{e}_1 + \vec{e}_2 \end{cases}$ bazisdagi chiziqli operator matritsasini toping.

Yechish. O‘tish matritsasi $C = \begin{pmatrix} 1 & 2 \\ -2 & 1 \end{pmatrix}$, unga teskari matritsa $C^{-1} = \frac{1}{5}\begin{pmatrix} 1 & -2 \\ 2 & 1 \end{pmatrix}$. Demak, yangi bazisda operatorning matritsasi quyidagi ko‘rinishda bo‘ladi:

$$A^* = C^{-1}AC = \frac{1}{5}\begin{pmatrix} 1 & -2 \\ 2 & 1 \end{pmatrix}\begin{pmatrix} 17 & 6 \\ 6 & 8 \end{pmatrix}\begin{pmatrix} 1 & 2 \\ -2 & 1 \end{pmatrix} = \begin{pmatrix} 5 & 0 \\ 0 & 20 \end{pmatrix}.$$

9- ta’rif. Agar A chiziqli operator va λ son uchun $\tilde{A}(x) = \lambda x$ tenglik o‘rinli bo‘lsa, u holda λ son $\tilde{A}(x)$ operatorning *xos soni*, unga mos $\vec{x}$ vektorga esa operatorning *xos vektori* deb ataladi.

Yuqoridagi tenglikni operatorning matritsasidan foydalanib yozsak, u holda quyidagi tenglamalar sistemasini hosil qilamiz:

$$\left.\begin{array}{l} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = \lambda \cdot x_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n = \lambda \cdot x_2 \\ ---------------- \\ a_{n1}x_1 + a_{n2}x_2 + \cdots + a_{nn}x_n = \lambda \cdot x_n \end{array}\right\} \Rightarrow \left.\begin{array}{l} (a_{11} - \lambda)x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = 0 \\ a_{21}x_1 + (a_{22} - \lambda)x_2 + \cdots + a_{2n}x_n = 0 \\ ---------------- \\ a_{n1}x_1 + a_{n2}x_2 + \cdots + (a_{nn} - \lambda)x_n = 0 \end{array}\right\}$$

Bundan $\left[A - \lambda E\right] \cdot X = 0$.

Bizga ma‘lumki bir jinsli chiziqli tenglamalar sistemasi har doim trivial yechimga ega. Chiziqli tenglamalar sistemasi trivial bo‘lmagan yechimga ega bo‘lishi uchun esa uning koeffitsiyentlaridan tuzilgan determinantning qiymati nolga teng bo‘lishi zarur va yetarli, ya‘ni

$$|A-\lambda E|=\begin{vmatrix} a_{11}-\lambda & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22}-\lambda & \dots & a_{2n} \\ \vdots & \vdots & \dots & \vdots \\ a_{n1} & a_{n2} & \dots & a_{nn}-\lambda \end{vmatrix}=0 \qquad \textbf{(6)}$$

$|A-\lambda E|$ determinant λ ga nisbatan n darajali ko‘phaddir. Bu ko‘phad $\tilde{A}(x)$ operatorning xarakteristik ko‘phadi deb ataladi. (6) tenglama $\tilde{A}(x)$ operatorning xarakteristik tenglamasi deyiladi. Chiziqli operatorning xarakteristik ko‘phadi bazisni tanlashga bog‘liq emas.

6- misol. $\tilde{A}(\vec{x})=\left(2x_1-x_2+2x_3,\, 5x_1-3x_2+3x_3,\, -x_1-2x_3\right)$ operatorning xos soni va xos vektorlarini toping.

Yechish. Avval $\tilde{A}$ operatorning matritsasini tuzib olamiz:

$$A=\begin{pmatrix} 2 & -1 & 2 \\ 5 & -3 & 3 \\ -1 & 0 & -2 \end{pmatrix}.$$

Berilgan operatorga mos keluvchi bir jinsli tenglamalar sistemasi quyidagi ko‘rinishni oladi:

$$\begin{cases} (2-\lambda)x_1-x_2+2x_3=0 \\ 5x_1-(3+\lambda)x_2+3x_3=0 \\ -x_1-(2+\lambda)x_3=0. \end{cases}$$

Bundan xarakteristik ko‘phadni topamiz:

$$p(\lambda)\equiv\begin{vmatrix} 2-\lambda & -1 & 2 \\ 5 & -3-\lambda & 3 \\ -1 & 0 & -2-\lambda \end{vmatrix}=-(\lambda+1)^3.$$

Demak, xos son $\lambda=-1$ ekan. Bu sonni sistemaga qo‘ysak,

$$\begin{cases} 3x_1-x_2+2x_3=0, \\ 5x_1-2x_2+3x_3=0, \\ -x_1-x_3=0. \end{cases}$$

Bundan $x_1 = x_2$, $x_1 = -x_3$. Demak, $X = \alpha \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$.

7- misol. Ushbu

$$A = \begin{pmatrix} 7 & -2 & 0 \\ -2 & 6 & -2 \\ 0 & -2 & 5 \end{pmatrix}$$

matritsaning xos soni va xos vektorlarini toping.

Yechish. Xarakteristik tenglamani tuzib yechamiz:

$$\begin{vmatrix} 7-\lambda & -2 & 0 \\ -2 & 6-\lambda & -2 \\ 0 & -2 & 5-\lambda \end{vmatrix} = 0;$$

$$-\lambda^3 + 18\lambda^2 - 99\lambda + 162 = 0,$$

$$\lambda_1 = 3,\ \lambda_2 = 6,\ \lambda_3 = 9.$$

$\lambda_1 = 3$ xos son uchun xos vektor

$$\left.\begin{aligned} 4x_1 - 2x_2 &= 0 \\ -2x_1 + 3x_2 - 2x_3 &= 0 \\ -2x_2 + 2x_3 &= 0 \end{aligned}\right\}$$

tenglamalar sistemasidan aniqlanadi. $x_1 = m$, deb qabul qilib, $x_2 = 2m,\ x_3 = 2m$ ni hosil qilamiz. Xos vektor: $\vec{\tau}_1 = m\vec{i} + 2m\vec{j} + 2m\vec{k}$. Shunga o‘xshash $\vec{\tau}_2 = m\vec{i} + \frac{1}{2}m\vec{j} - m\vec{k}$; $\vec{\tau}_3 = -m\vec{i} + m\vec{j} - \frac{1}{2}m\vec{k}$ xos vektorlarni topamiz.

8- misol. Agar R^3 da chiziqli A operator $\vec{e_1}, \vec{e_2}, \vec{e_3}$ bazisda o‘zining $A = \begin{pmatrix} 3 & 2 & 4 \\ -1 & 5 & 6 \\ 1 & 8 & 2 \end{pmatrix}$ matritsasi bilan berilgan bo‘lsa, $\vec{x} = 4\vec{e_1} - 3\vec{e_2} + \vec{e_3}$ vektorning $\vec{y} = A(\vec{x})$ aksini toping.

Yechish. $Y = AX$ formulaga binoan, $\begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \begin{pmatrix} 3 & 2 & 4 \\ -1 & 5 & 6 \\ 1 & 8 & 2 \end{pmatrix} \cdot \begin{pmatrix} 4 \\ -3 \\ 1 \end{pmatrix} = \begin{pmatrix} 10 \\ -13 \\ -18 \end{pmatrix}$

Demak, $\vec{y} = 10\vec{e_1} - 13\vec{e_2} - 18\vec{e_3}$

9- misol. $\vec{e_1},\ \vec{e_2}$ bazisda A operator $A=\begin{pmatrix}17 & 6\\ 6 & 8\end{pmatrix}$ matritsaga ega. $\begin{cases}\vec{e_1'}=\vec{e_1}-2\vec{e_2}\\ \vec{e_2'}=2\vec{e_1}+\vec{e_2}\end{cases}$ bazisda A operatorning matritsasini toping.

Yechish. O‘tish matritsasi $C=\begin{pmatrix}1 & 2\\ -2 & 1\end{pmatrix}$ ning teskari matrisasi $C^{-1}=\frac{1}{5}\begin{pmatrix}1 & -2\\ 2 & 1\end{pmatrix}$ Demak,

$$B=C^{-1}AC=\frac{1}{5}\begin{pmatrix}1 & -2\\ 2 & 1\end{pmatrix}\cdot\begin{pmatrix}17 & 6\\ 6 & 8\end{pmatrix}\cdot\begin{pmatrix}1 & 2\\ -2 & 1\end{pmatrix}=\begin{pmatrix}1 & -2\\ 8 & 4\end{pmatrix}\cdot\begin{pmatrix}1 & 2\\ -2 & 1\end{pmatrix}=\begin{pmatrix}5 & 0\\ 0 & 20\end{pmatrix}$$

10- misol. $\vec{e_1},\ \vec{e_2}$ bazisda chiziqli operatorning matritsasi $A=\begin{pmatrix}2 & 3\\ 5 & 4\end{pmatrix}$ ko‘rinishga ega. Yangi $\begin{cases}\vec{e_1'}=5\vec{e_1}-3\vec{e_2}\\ \vec{e_2'}=\vec{e_1}+2\vec{e_2}\end{cases}$ bazisda chiziqli operatorning matritsasini toping.

11- misol. $\vec{e_1},\vec{e_2},\vec{e_3}$ bazisda chiziqli operatorning matritsasi $A=\begin{pmatrix}0 & 1 & 2\\ -4 & 3 & 0\\ 2 & 1 & 2\end{pmatrix}$ ko‘rinishda. Yangi $\begin{cases}\vec{e_1'}=2\vec{e_1}+2\vec{e_2}+2\vec{e_3}\\ \vec{e_2'}=\vec{e_1}-\vec{e_2}+\vec{e_3}\\ \vec{e_3'}=\vec{e_1}+\vec{e_2}-\vec{e_3}\end{cases}$ bazisda A operatorning matritsasini toping.

12- misol. $A(\vec{x})=(2x_1+x_3;\ 4x_2-2x_3;\ 3x_1+x_2-x_3)$ operatorni chiziqlilikka tekshiring.

Yechish. Operatorni chiziqlilikka tekshirish uchun $A(\vec{x}+\vec{y})=A(\vec{x})+A(\vec{y})$ hamda $A(\alpha\vec{x})=\alpha A(\vec{x})$ tengliklarni bajarilishini tekshirish kifoya.

$$A(\vec{x}+\vec{y})=\begin{pmatrix}2(x_1+y_1)+x_3+y_3\\ 4(x_2+y_2)-2(x_3+y_3)\\ 3(x_1+y_1)+x_2+y_2-(x_3+y_3)\end{pmatrix}=\begin{pmatrix}2x_1+x_3+2y_1+y_3\\ 4x_2-2x_3+4y_2-2y_3\\ 3x_1+x_2-x_3+3y_1+y_2-y_3\end{pmatrix}=$$

$$=\begin{pmatrix} 2x_1+x_3 \\ 4x_2-2x_3 \\ 3x_1+x_2-x_3 \end{pmatrix}+\begin{pmatrix} 2y_1+y_3 \\ 4y_2-2y_3 \\ 3y_1+y_2-y_3 \end{pmatrix}=A\left(\vec{x}\right)+A\left(\vec{y}\right)$$

$$A\left(\alpha\vec{x}\right)=\begin{pmatrix} 2\alpha x_1+\alpha x_3 \\ 4\alpha x_2-2\alpha x_3 \\ 3\alpha x_1+\alpha x_2-\alpha x_3 \end{pmatrix}=\alpha\begin{pmatrix} 2x_1+x_3 \\ 4x_2-2x_3 \\ 3x_1+x_2-x_3 \end{pmatrix}=\alpha A\left(\vec{x}\right)$$

13- misol Chizqli A operator $A=\begin{pmatrix} 1 & 4 \\ 9 & 1 \end{pmatrix}$ matritsa bilan berilgan. Chiziqli operatorning hos qiymatlari va hos vektorlarini toping.

Yechish. Xarakteristik tenglama tuzamiz: $|A-\lambda E|=\begin{vmatrix} 1-\lambda & 4 \\ 9 & 1-\lambda \end{vmatrix}=0$

$\lambda^2-2\lambda-35=0,\ \lambda_1=-5,\ \lambda_2=7$

$\lambda_1=-5$ xos qiymatga mos $\overrightarrow{X}^{(1)}=\left(x_1;\ x_2\right)$ xos vektorni topamiz. Buning uchun quyidagi tenglamani yechamiz:

$$\lambda_1=-5,\ \left(A-\lambda E\right)\cdot\vec{x}=\theta,\ \begin{pmatrix} 6 & 4 \\ 9 & 6 \end{pmatrix}\cdot\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}=\begin{pmatrix} 0 \\ 0 \end{pmatrix},\ x_2=-1{,}5x_1$$

Agar $x_1=C$ deb olsak, $x_2=-1{,}5C$ bo‘ladi. Demak $\vec{x}^{(1)}=\left(c;\ -1.5c\right)$ vector A operatorning $\lambda_1=-5$ xos qiymatiga mos xos vector bo‘ladi. Xuddi shunga o‘xshab $\lambda_2=7$ xos qiymatga mos xos vektorlarni $\vec{x}^{(2)}=\left(\frac{2}{3}C_1;\ C_1\right),\ C_1\neq 0$ aniqlash mumkin.