Mavzu-8. Tekislikda to'g'ri chiziqning umumiy tenglamasi va uning turli xususiy ko'rinishlari. Tekislikda 2- tartibli chiziqlar. Aylana, ellips, giperbola, parabola.

Reja:

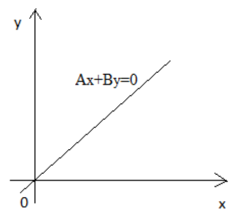
1. Tekislikda to'g'ri chiziq tenglamasi
2. To'g'ri chiziqning kesmalardagi tenglamasi
3. Tekislikda to'g'ri chiziqning normal tenglamasi
4. Tekislikda 2- tartibli chiziqlar.

Tekislikda har qanday to'g'ri chiziq Ax+By+C=0 tenglama bilan aniqlanadi.

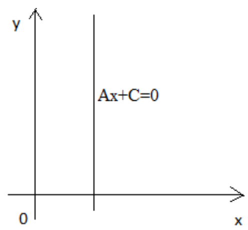
A, B, C, lar to'g'ri chiziqning koeffitsiyentlari bo'lib, ular turli qiymatlarda turli chiziqlar hosil bo'ladi. Demak, to'g'ri chiziqning tekislikdagi vaziyati shu A, B, C, lar bilan to'liq aniqlanadi.

Xususiy hollarni ko'rib chiqamiz.

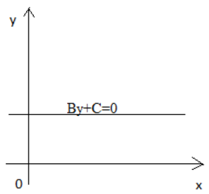
1) B=C=0 va A≠0 bo'lsa, Ax=0 to'g'ri chiziq hosil bo'ladi. U O_y o'qi bilan ustma-ust tushadi.
2) A=C=0 va B≠0 bo'lsa By=0 O_x o'qi bilan ustma-ust tushadi.
3) C=0; A≠0; B≠0 bo'lsa, Ax+By=0 koordinatalar boshidan o'tadi.



4) B=0; A≠0; C≠0 bo'lsa, Ax+C=0 O_y o'qiga parallel o'tadi.

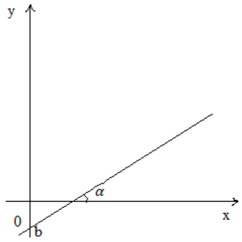


5) A=0; B≠0; C≠0 bo'lsa, By+C=0 O_x o'qiga parallel o'tadi



<u>To'g'ri chiziqning burchak koeffitsienti tenglamasi</u>

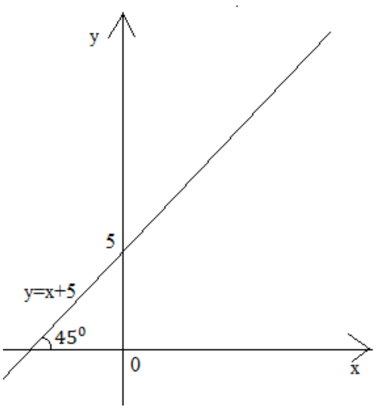
Agar Ax+By+C=0 tenglamada B≠0 bo'lsa, $y=-\frac{A}{C}x-\frac{B}{C}$ bo'ladi $\Rightarrow$ y=kx+b to'g'ri chiziqning burchak koeffitsienti tenglamasi.



k=tg α to'g'ri chiziqning burchak koeffitsienti deyiladi.

b-O_y o'qidan ajratgan kesma uzunligi;

Misol y=x+5 to'g'ri chiziqning O_x o'qi bilan tashkil etgan burchagini toping.



k=1 tgα =1 α =45

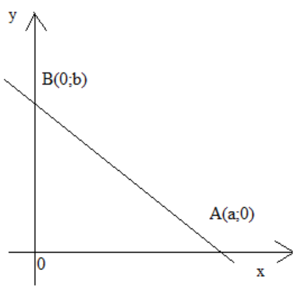
To'g'ri chiziqning kesmalardagi tenglamasi

Ax+By+C=0 umumiy tenglamada C≠0 bo'lsa, hamma hadini – C ga bo'lib yuborsak,

$-\frac{A}{C}x-\frac{B}{C}=1$, $\frac{x}{a}+\frac{y}{b}=1$ (bu yerda $-\frac{A}{C}=\frac{1}{a}$, $-\frac{B}{C}=\frac{1}{b}$) to'g'ri chiziqning kesmalardagi tenglamasi deyiladi.

Bu yerda a-O_x o'qidan ajratgan kesma

b-O_y o'qidan ajratgan kesma



Misol 2x+5y+3=0tenglama bilan berilgan to'g'ri chiziq O_x va O_y o'qidan qanday kesma ajratadi?

2x+5y+3=0 c=3 -c=-3 ga bo'lib olamiz.

$-\frac{2}{3}x-\frac{5}{3}y=1$, $\frac{x}{-\frac{3}{2}}+\frac{y}{-\frac{3}{5}}=1$ $a=-\frac{3}{2}$ - Ox o'qidan ajratilgan kesma;

$b=-\frac{3}{5}$ - Oy o'qidan;

Berilgan ikki nuqtadan o'tuvchi to'g'ri chiziq tenglamasi

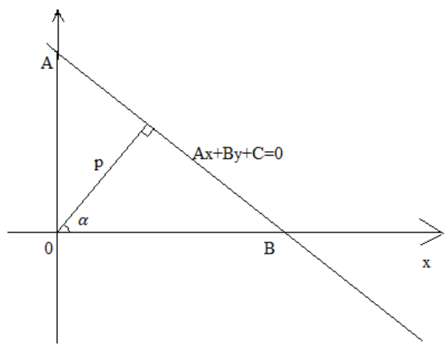
Ikkita $M_1(x_1;y_1)$ va $M_2(x_2;y_2)$ nuqtalardan o'tuvchi to'g'ri chiziq tenglamasi

$\frac{x-x_1}{x_2-x_1}=\frac{y-y_1}{y_2-y_1}$ kabi yoziladi.

Misol M(4:1) va N (5:7) nuqtalardan o'tuvchi to'g'ri chiziq umumiy tenglamasini yozing

$\frac{x-4}{5-4}=\frac{y-1}{7-1}$, $\frac{x-4}{1}=\frac{y-1}{6}$, 6x-24=2y-1 , 6x-y-23=0

Tekislikda to'g'ri chiziqning normal tenglamasi



$x\cos\alpha + y\sin\alpha - p = 0$

Ax+By+C=0 umumiy tenglamaning ikki qismini ham $\mu \neq 0$ songa ko'paytiramiz,

μ Ax+ μ By+ μ C=0 ; u holda

$\mu A=\cos\alpha$, $\mu B=\sin\alpha$, $\mu C=-p$ bo'ladi.

$(\mu A)^2+(\mu B)^2=\cos^2\alpha+\sin^2\alpha=1$

$\mu^2=\left(A^2+B^2\right)=1$, $\mu^2=\frac{1}{A^2+B^2}$, $\mu=\pm\sqrt{\frac{1}{A^2+B^2}}$

$\cos\alpha=\pm\frac{A}{\sqrt{A^2+B^2}}$, $\sin\alpha=\pm\frac{B}{\sqrt{A^2+B^2}}$, $-p=\pm\frac{C}{\sqrt{A^2+B^2}}$

natijada Ax+By+C=0 tenglama

$\pm\frac{A}{\sqrt{A^2+B^2}}x\pm\frac{B}{\sqrt{A^2+B^2}}y\pm\frac{C}{\sqrt{A^2+B^2}}=0$ ko'rinishga keladi.

Misol 3x+4y+2=0 to'g'ri chziqni normal tenglamasini toping.

A=3 , B=4 , C=2

$\pm\sqrt{A^2+B^2}=\pm\sqrt{9+16}=\pm\sqrt{25}=\pm5$ (-5 ni olamiz, C ning ishorasiga qarshi)

$-\frac{3}{5}x-\frac{4}{5}y-\frac{2}{5}=0$

To'g'ri chiziqning parallellik va perpendikularlik sharti

$y=k_1x+b_1$, $y=k_2x+b_2$ kabi berilgan bo'lsin

$k_1 = k_2$ $\quad\|\quad$ parallellik sharti

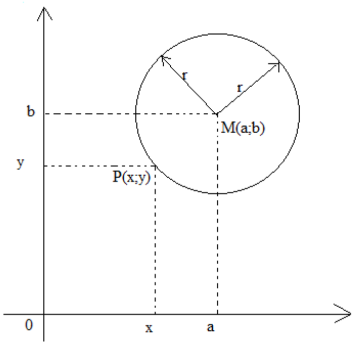
$k_1 = -\dfrac{1}{k_2}$ $\quad\perp\quad$ perpendikularlik sharti.

2-tartibli egri chiziqlar – bular aylana, ellips, giperbola va parabolalardir.

Aylana

__**Ta’rif-1**__ Berilgan M(a;b) nuqtadan bir xil r masofada joylashgan nuqtalarning geometrik o’rni aylana deyiladi. M(a;b) nuqta aylana markazi, r- radiusi.

Aylanadan ixtiyoriy P(x;y) nuqta olamiz. P(x;y) va M(a,b) nuqtalar orasidagi masofa $\sqrt{(x-a)^2+(y-b)^2}=r$ Kvadratga oshirsak $(x-a)^2+(y-b)^2=r^2$ Bu markazi (a;b) nuqtada, radiusi r bo’lgan aylana tenglamasi. Xususan, markazi koordinatalar boshida bo’lganda: $x^2+y^2=r^2$ ko’rinishga keladi.

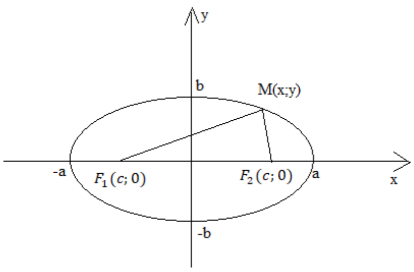


Misol Markazi (3;4) nuqtada, radiusi 5 ga teng aylana tenglamasini yozing.

a=3 , b=4 , r=5 $\quad (x-3)^2+(y-4)^2=5^2$

Ellips

Tekisliklarda $F_1=(a_1;b_1)$ va $F_2=(a_2;b_2)$ nuqtalar berilgan bo’lsin.



__**Ta’rif-2.**__ F_1 va F_2 nuqtalargacha bo’lgan masofalarning yig’indisi o’zgarmas bo’lgan nuqtalarning geometrik o’rni __**ellips**__ deyiladi. F_1 va F_2 -ellipsning fokuslari.

Ikki nuqta orasidagi masofa formulasiga ko’ra

$$\sqrt{(x-c)^2+y^2}+\sqrt{(x+c)^2+y^2}=2a$$

$$\left(\sqrt{(x-c)^2+y^2}\right)^2=\left(2a-\sqrt{(x+c)^2+y^2}\right)^2$$

$$(x-c)^2+y^2=4a^2-4a\sqrt{(x+c)^2+y^2}+2xc$$

$$4a\sqrt{(x+c)^2+y^2}=4a^2+4xc$$

$$a\sqrt{(x+c)^2+y^2}=a^2+xc$$

$$a^2\left((x+c)^2+y^2\right)=a^4+2a^2cx+c^2x^2$$

$$a^2x^2+2cxa^2+a^2c^2+a^2y^2=a^2(a^2+2cx)+c^2x^2$$

$$a^2x^2+a^2c^2+a^2y^2=a^2a^2+c^2x^2$$

$x^2\left(a^2-c^2\right)+a^2y^2=a^2\left(a^2-c^2\right)$, $a^2-c^2=b^2$ deb olsak,

$x^2b^2+a^2y^2=a^2b^2$, a^2b^2 ga bo'lib:

$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ **ellipsning kanonik tenglamasi** ni hosil qilamiz.

$e=\frac{c}{a}$ miqdor ellipsning **<u>ekssentrisiteti</u>** deyiladi

<u>Misol.</u> $\frac{x^2}{100}+\frac{y^2}{36}=1$ tenglama bilan berilgan ellipsning ekssentrisitetini toping.

a=10 b=6 $a^2-c^2=b^2$ $c^2=100-36=64$ c=8 $e=\frac{c}{a}=\frac{8}{10}=0,8$

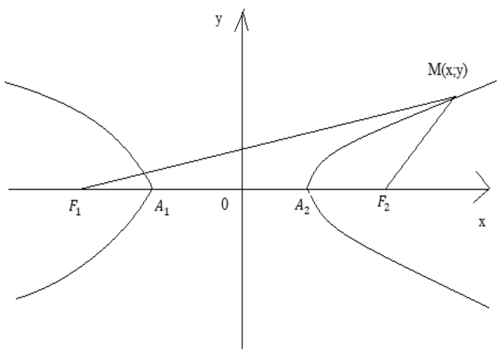
<u>Giperbola</u>

Tekislikda F_1 va F_2 nuqtalar berilgan bo'lsin.

<u>Ta'rif-3</u> F_1 va F_2 nuqtalargacha bo'lgan masofalar ayirmasi o'zgarmas bo'lgan nuqtalarning geometrik o'rniga **giperbola** deyiladi.

F_1, F_2 – giperbolaning fokuslari

A_1, A_2 – giperbolaning uchlari



$\sqrt{(x-c)^2+y^2}+\sqrt{(x+c)^2+y^2}=\pm2a$ xuddi oldingidek hisob-kitoblardan so'ng,

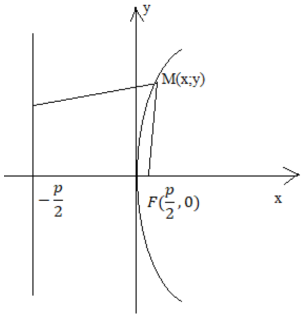
$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ giperbolaning kanonik tenglamasi.

$e = \frac{c}{a}$ giperbolaning ekssentrisiteti deyiladi.

Parabola

Dekart koordinatalar tekisligida O_y o'qiga parallel to'g'ri chiziq va bu to'g'ri chiziqqa tegishli bo'lmagan F(a;b) nuqta berilgan bo'lsin.

Ta'rif-4 To'g'ri chiziq va F nuqtadan bir xil masofada joylashgan nuqtalarning geometrik o'rniga **parabola** deyiladi.



F-parabola fokusi

$-\frac{p}{2}$ to'g'ri chiziq direktrisasi

$\sqrt{\left(x - \frac{p}{2}\right)^2 + y^2} = x + \frac{p}{2}$ kvadratga oshirib parabolaning kanonik tenglamasini topamiz

$y^2 = 2px$ **Parabolaning kanonik tenglamasi**

Takrorlash uchun savollar:

1. Kesmada to'g'ri chiziq tenglamasi.
2. Ikki nuqtadan o'tuvchi to'g'ri chiziq tenglamasi.
3. To'g'ri chziqning normal tenglamasi.
4. To'g'ri chiziqning parallellik va penpendikularlik sharti.
5. Ellips deb nimaga aytiladi?
6. Aylanaga ta'rif bering.
7. Giperbolaning kanonik tenglamasini ayting.
8. Parabolaning kanonik tenglamasini ayting.

Foydalanilgan adabiyotlar:

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3. www.mathprofi.ru internet sayti.