Mavzu-4. Matritsalar va ularning ayrim xossalari. Matritsalar ustida amallar. Determinantlar. Matritsaning determinant. Minor va algebraik to'ldiruvchilar. Teskari matritsalar. Ixtiyoriy tartibli determinantlarni hisoblash.

Reja:

1. Matritsa tushunchasi.
2. Matritsa xossalari.
3. Matritsalar ustida amallar.
4. Determinant tushunchasi.
5. Minor va algebraik to'ldiruvchi.
6. n- tartibli determinantlarni hisoblash.

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + ... + a_{1n}x_n = c_1 \\ a_{21}x_1 + a_{22}x_2 + ... + a_{2n}x_n = c_2 \\ .... \\ a_{m1}x_1 + a_{m2}x_2 + ... + a_{mn}x_n = c_m \end{cases} \quad (1)$$

Chiziqli m ta tenglamadan iborat sistema berilgan bo'lsin. Noma'lum x lar oldidagi koeffitsientlardan tuzilgan ushbu jadvalga matritsa deyiladi:

$$\begin{pmatrix} a_{11}, a_{12}, ..., a_{1n} \\ a_{21}, a_{22}, ..., a_{2n} \\ .... \\ a_{m1}, a_{m2}, ..., a_{mn} \end{pmatrix}$$ m*n – tartibli matritsa m- ta satr va n- ta ustundan iborat bo'ladi:

a_{ij} - sonlar matritsa elementlari deyiladi *(i=1,m ; j=1,n)*

Masalan, a_{34} - elelement 3- satr va 4- ustunda joylashganini bildiradi. Matritsalar lotin alifbosining bosh harflari A,B,C.. lar bilan belgilanadi: $A = \left\| a_{ij} \right\|$, i=1,m ; j=1,n

Agar matritsaning barcha elementlari 0 ga teng bo'lsa, u nol matritsa deyiladi:

$$0 = \begin{Vmatrix} 0, 0, ..., 0 \\ 0, 0, ..., 0 \\ .... \\ 0, 0, ..., 0 \end{Vmatrix}$$

Agar matritsaning ustunlari va satrlar soni teng bo'lsa, ya'ni m=n bo'lsa, u holda n- tartibli kvadrat matritsa hosil bo'ladi:

$$\begin{Vmatrix} a_{11}, a_{12}, ..., a_{1n} \\ a_{21}, a_{22}, ..., a_{2n} \\ .... \\ a_{n1}, a_{n2}, ..., a_{nn} \end{Vmatrix}$$ $a_{11}, a_{22}, ..., a_{nn}$ - elementlari bosh diagonal elementlari deyiladi.

Agar kvadrat matritsaning bosh diogonalidan boshqa barcha elementlari nol bo'lsa, u diogonal matritsa deyiladi:

$$\begin{Vmatrix} a_{11}, 0, ..., 0 \\ 0, a_{22}, ..., 0 \\ .... \\ 0, 0, ..., a_{nn} \end{Vmatrix}$$

Agar diagonal matritsada $a_{11} = a_{22} = ... = a_{nn} = 1$ bo'lsa:

$$\begin{Vmatrix} 1, 0, ..., 0 \\ 0, 1, ..., 0 \\ .... \\ 0, 0, ..., 1 \end{Vmatrix} = E$$ bo'lsa u holda <u>birlik matritsa</u> deyiadi va *E* bilan belgilanadi.

A kvadrat matritsaning satrlarini ustunlari bilan almashtirish orqali hosil qilingan matritsa <u>transponirlangan matritsa</u> deyiladi va A' bilan belgilanadi:

$$A = \begin{Vmatrix} a_{11}, a_{12}, ..., a_{1n} \\ a_{21}, a_{22}, ..., a_{2n} \\ .... \\ a_{n1}, a_{n2}, ..., a_{nn} \end{Vmatrix}, \qquad A' = \begin{Vmatrix} a_{11}, a_{21}, ..., a_{n1} \\ a_{12}, a_{22}, ..., a_{n2} \\ .... \\ a_{1n}, a_{2n}, ...a_{nn} \end{Vmatrix}$$

Agar *A* kvadrat matritsa transponirlangan A' matritsaga teng bo'lsa, ya'ni *A*= A' bo'lsa u holda A – simmetrik matritsa deyiladi.

Agar ikkita *A* va *B* matritsalarning har bir mos $a_{ij} = b_{ij}$ elementlari o'zaro teng bo'lsa, bunday matritsalar o'zaro teng deyiladi va A=B kabi yoziladi.

Ikkita m*n tartibli matritsalar berilgan bo'lsin:

$$A = \begin{Vmatrix} a_{11}, a_{12}, ..., a_{1n} \\ a_{21}, a_{22}, ..., a_{2n} \\ .... \\ a_{n1}, a_{n2}, ..., a_{nn} \end{Vmatrix}, \qquad B = \begin{Vmatrix} b_{11}, b_{12}, ..., b_{1n} \\ b_{21}, b_{22}, ..., b_{2n} \\ .... \\ b_{n1}, b_{n2}, ..., b_{nn} \end{Vmatrix}$$

Bu matritsalarning mos elementlari yig'indisidan tuzilgan m*n tartibli *A* va *B* matritsalar yig'indisi deyiladi va *A*+*B* kabi belgilanadi:

$$A + B = \begin{Vmatrix} a_{11} + b_{11}, a_{12} + b_{12}, ..., a_{1n} + b_{1n} \\ a_{21} + b_{21}, a_{22} + b_{22}, ..., a_{2n} + b_{2n} \\ .... \\ a_{m1} + b_{m1}, a_{m2} + b_{m2}, ..., a_{mn} + b_{mn} \end{Vmatrix}$$

Xuddi shuningdek, mos elementlar ayirmasidan tuzilgan m*n- o'lchamli matritsaga *A* va *B* matritsalar ayirmasi deyiladi va *A*-*B* kabi belgilanadi.

Biror λ son va *A* berilgan bo'lsin. *A* matritsaning har bir elementini λ songa ko'paytirishdan hosil bo'lgan matritsaga λ son va *A* matritsaning <u>ko'paytmasi</u> deyiladi va λA kabi belgilanadi:

Demak, $$\lambda A = \begin{Vmatrix} \lambda a_{11}, \lambda a_{12}, ..., \lambda a_{1n} \\ \lambda a_{21}, \lambda a_{22}, ..., \lambda a_{2n} \\ .... \\ \lambda a_{m1}, \lambda a_{m2}, ..., \lambda a_{mn} \end{Vmatrix}$$

Xossalari:

1. *A+0=0+A=A*
2. *A+B=B+A*

3. $\lambda(\mu A) = (\lambda\mu)A$
4. $\lambda(A + B) = \lambda A + \lambda B$
5. $(\lambda + \mu)A = \lambda A + \mu A$

Misol-1 Agar $A = \begin{Vmatrix} 2,.4,.1 \\ -1,0,2 \end{Vmatrix}$, $B = \begin{Vmatrix} 0,2,1 \\ 1,1,2 \end{Vmatrix}$ bo'lsa, *A+B*, *A-B*, *2A-3B* matritsalarni toping

$A+B = \begin{Vmatrix} 2,6,2 \\ 0,1,4 \end{Vmatrix}$, $A-B = \begin{Vmatrix} 2.,.2.,.0 \\ -2,-1,0 \end{Vmatrix}$, $2A = \begin{Vmatrix} 4,.8,.2 \\ -2,0,4 \end{Vmatrix}$, $3B = \begin{Vmatrix} 0,6,3 \\ 3,3,6 \end{Vmatrix}$, $2A-3B = \begin{Vmatrix} 4,.2.,.-1 \\ -5,-3,-2 \end{Vmatrix}$

$A = \begin{Vmatrix} a_{11}, a_{12}, ..., a_{1n} \\ a_{21}, a_{22}, ..., a_{2n} \\ .... \\ a_{m1}, a_{m2}, ..., a_{mn} \end{Vmatrix}$, $B = \begin{Vmatrix} b_{11}, b_{12}, ..., b_{1n} \\ b_{21}, b_{22}, ..., b_{2n} \\ .... \\ b_{m1}, b_{m2}, ..., b_{mn} \end{Vmatrix}$ lar berilgan bo'lsin.

$$A*B = \begin{Vmatrix} d_{11}, d_{12}, ..., d_{1n} \\ d_{21}, d_{22}, ..., d_{2n} \\ .... \\ d_{m1}, d_{m2}, ..., d_{mn} \end{Vmatrix}$$

$$d_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + ... + a_{in}b_{nj} = \sum_{k=1}^{n} a_{ik}b_{kj} \qquad (i, j = 1, 2, 3, ..., n)$$

*A*B≠B*A* o'rin almashtirish xossasi o'rinli emas !

Biroq *E* birlik matritsa uchun *AE=EA=A*

A,*B*, va *C* matritsalaar berilgan bo'lsin, u holda:

6. *(A+B)*C=AC+BC*
7. *(A*B)*C=A*(B*C)*

Biror a,b,c,d sonlardan tuzilgan A matritsa berilgan bo'lsin:

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

Ta'rif-1 Ushbu $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$ ifoda 2-tartibli determinant deyiladi va ***detA*** yoki *Δ* bilan ifodalanadi.

det A= $\Delta = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$ =ad-bc ayirma esa uning qiymati deyiladi $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$

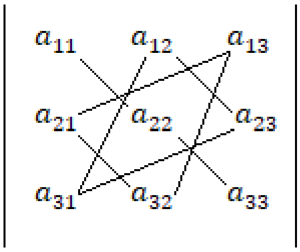
a va d bosh dioganal, c va b esa yordamchi dioganal elementlari deyiladi.

Misol: $A = \begin{vmatrix} 3 & 5 \\ 2 & 4 \end{vmatrix}$ determinantni hisoblang.

d*et A*= 4*3-2*5=12-10=2

3-tartibli determinant berilgan bo'lsin.

A=

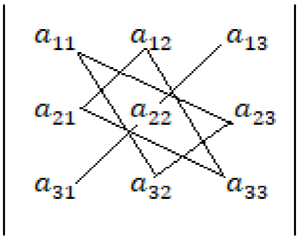


(1)

Ushbu 3-tartibli ushbu determinant quyidagicha hisoblanadi.

det A = $a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{21}a_{13}a_{32} - a_{13}a_{22}a_{31} - a_{21}a_{12}a_{33} - a_{32}a_{23}a_{11}$

A=



(2)

Determinant 6 haddan iborat bo’lib, dastlabki 3 ta musbat hadi (1) sxema bo’yicha keying 3 ta manfiy hadi (2) sxema bo’yicha aniqlanadi.

Misol: $A = \begin{vmatrix} 2 & 1 & 3 \\ 4 & 2 & 5 \\ 2 & 1 & 6 \end{vmatrix}$ determinantni hisoblang.

det A=2*2*6+1*5*2+4*1*3-3*3*2-1*5*2-4*1*6=0

Ta’rif-2: Ixtiyoriy determinant uchun *Δ*=0 bo’lsa xos, *Δ* ≠0 bo’lsa xosmas deyiladi.

Yuqori tartibli determinantlarni hisoblash uchun determinantning xossalari bilan tanishib chiqaylik.

XOSSALARI

Biror $\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$ uchinchi tartibli determinant berilgan bo’lsin.

1) Determinantning biror yo’lini unga mos ustuni bilan almashtirilsa, determinant qiymati o’zgarmaydi.

2) Determinantning ixtiyoriy ikki satri (ustuni) o’rnini o’zaro almashtirsak, determinant qiymati o’zgarmaydi, ishorasi qarama-qarshiga o’zgaradi.

Natija-1 Determinantning ixtiyoriy ikki satri (ustuni) bir xil bo'lsa, determinant qiymati nol bo'ladi.

3) Determinantning ixtiyoriy satri (ustuni)dagi barcha elementlarni biror o'zgarmas ***k*** songa ko'paytirilsa determinant qiymati ham ***k*** ga ko'payadi

$$k\Delta = \begin{vmatrix} ka_{11} & ka_{12} & ka_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

4) Determinantning biror yo'li yoki ustunidagi barcha elementlari nol bo'lsa, determinant qiymati $\Delta = 0$

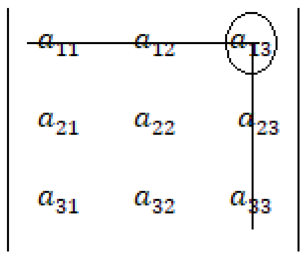
5) Determinantning ixtiyoriy ikkita satri yoki ustuni o'zaro proporsional bo'lsa, determinant $\Delta = 0$ bo'ladi.

6) Agar determinantning biror satri yoki ustunidagi elementlar ikki qo'shiluvchining yig'indisidan iborat bo'lsa:

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21}+b_1 & a_{22}+b_2 & a_{23}+b_3 \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} + \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ b_1 & b_2 & b_3 \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$ bo'ladi

Natija-2. Agar determinantning biror satri (yoki ustuni)dagi barcha elementlarni biror o'zgarmas ***k*** songa ko'paytirib uni uni boshqa satri (yoki ustuni)ga qo'shilsa, determinant o'zgarmaydi.

Keyingi xossalarni ifodalash uchun minor va algebraik to'ldiruvchi tushunchasini kiritamiz.

A=


Berilgan A determinantda a_{13} element turgan 1-satr va 3-ustun elementlarini o'chirib hosil qilingan

$\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$ determinant a_{13} elementning minori deyiladi.

Tarif-3. a_{ij} elementga mos keluvchi **minor** deb shu element joylashgan *i*-satr va *j*-ustun elementlarini o'chirib hosil qilingan determinantga aytiladi va M_{ij} kabi belgilanadi.

Tarif-4. Ushbu $(-1)^{i+j} M_{ij}$ miqdor va a_{ij} elementga mos algebraik to'ldiruvchisi deyiladi va A_{ij} kabi belgilanadi.

$$A_{ij} = (-1)^{i+j} M_{ij}$$

Misol. Ushbu $A=\begin{vmatrix}4 & 3 & 1\\ 5 & 1 & 0\\ 1 & 2 & 7\end{vmatrix}$ determinantda a_{23} elementga mos keluvchi minor va algebraik to'ldiruvchini toping.

$A=\begin{vmatrix}4 & 3 & 1\\ 5 & 1 & 0\\ 1 & 2 & 7\end{vmatrix}$, $M_{23}=\begin{vmatrix}4 & 3\\ 4 & 2\end{vmatrix}$=8-12=-4 $A_{23}=(-1)^{2+3}M_{23}=(-1)*(-4)=4$

7) Determinantning biror satri yoki ustunida turgan barcha elementlarni ularga mos keluvchi algebraik to'ldiruvchilari bilan ko'paytmasidan tuzilgan yig'indi shu determinant qiymatiga teng.

$$A=\begin{vmatrix}a_{11} & a_{12} & a_{13}\\ a_{21} & a_{22} & a_{23}\\ a_{31} & a_{32} & a_{33}\end{vmatrix}=a_{11}A_{11}+a_{12}A_{12}+a_{13}A_{13}$$

8) Determinantning biror yo'li yoki (ustuni)da turgan barcha elementlarni boshqa yo'li (ustuni)da turgan elementlarga mos keluvchi algebraik to'ldiruvchilari bilan ko'paytmasidan tuzilgan yig'indi nolga teng bo'ladi:

$$a_{11}A_{11}+a_{12}A_{12}+a_{13}A_{13}=0$$

Ta'rif-5. *A* matritsaning noldan farqli minorlarining eng katta tartibiga uning ***rangi*** deyiladi va *rankA* kabi belgilanadi.

Misol: $A=\begin{vmatrix}2 & 3 & 5\\ 4 & 1 & 0\\ 2 & 3 & 5\end{vmatrix}$ matritsaning rangini toping

$\begin{vmatrix}2 & 3\\ 4 & 1\end{vmatrix}=2-12=-10$; $\begin{vmatrix}2 & 3 & 5\\ 4 & 1 & 0\\ 2 & 3 & 5\end{vmatrix}=0$

Shunday qilib noldan farqli minorlarining yuqori tartibi- 2 *rankA*=2

Biror n*n tartibli kvadrat matritsa berilgan bo'lsin $A=\begin{pmatrix}a_{11} & a_{12} & \dots & a_{1n}\\ a_{21} & a_{22} & \dots & a_{2n}\\ \dots & \dots & \dots & \dots\\ a_{n1} & a_{n2} & \dots & a_{nn}\end{pmatrix}$

Tarif-5. *AB*=*BA*=*E* tenglikni qanoatlantiruvchi *B*-matritsa *A* ga teskari matritsa deyiladi va A^{-1} kabi belgilanadi: $A*A^{-1}=E$

Takrorlash uchun savollar:

1. Transponirlangan matritsa, simmetrik matritsa nima?
2. A,B matritsalar va λ,μ- sonlari berilgan bo'lsa, bu matritsalar uchun 7 ta xossani aytib bering.
3. $A=\begin{Vmatrix}3,.1,.0\\2,1,-2\\3,4,-1\end{Vmatrix}$, $B=\begin{Vmatrix}-2,1,3\\4,-1,2\\5,.2,.0\end{Vmatrix}$ bo'lsa, *3A-2B=?*
4. $A=\begin{Vmatrix}4,.1,.0\\2,0,-1\\-2,1,3\end{Vmatrix}$, $B=\begin{Vmatrix}-1,1,2\\0,.1,.2\\2,.3,.1\end{Vmatrix}$ bo'lsa, *A*B=?*
5. Determinant nima?
6. Determinantning xossalari ayting.
7. Minor va algebraik to'ldiruvchilarni ta'riflang.
8. Teskari matritsa nima?

Foydalanilgan adabiyotlar:

1. Matematika. (A. Sanginov) 2011-y. Toshkent.
2. Курс математического анализа. (Л. Кудрявцев) 1998-г. Москва.
3. Oliy matematikadan misol va masalalar. (Sh.Xurramov) 2015-y. Toshkent.
4. www.mathprofi.ru internet sayti.