Mavzu-3. Kompleks sonlarni tasvirlash. Kompleks sonlarni moduli va argumenti. Kompleks sonlarni shakllari. Eyler va Muavr formulalari.

Reja:

1. Kompleks son tushunchasi.
2. Kompleks sonlarni moduli va argumenti.
3. Kompleks sonlarni shakllari.
4. Eyler va Muavr formulalari.

$x^2 + 1 = 0$ ko'rinishidagi tenglamalarni yechish haqiqiy sonlar to'plami R ni kengaytirish zaruratini keltirib chiqardi.

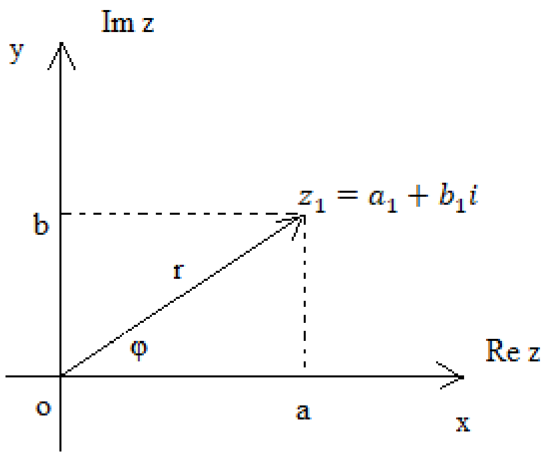
$x^2 + 1 = 0$; $x^2 = -1$; $x = \sqrt{-1}$; i=$\sqrt{-1}$; $i^2 = -1$; i- mavhum birlik.

Ta'rif-1 a+bi ifodaga kompleks son deyiladi, bu yerda a,b- haqiqiy sonlar, i- mavhum birlik, a- kompleks sonning haqiqiy qismi, bi- mavhum qismi deyiladi. Kompleks sonlar z harfi bilan belgilanadi: z=a+bi

Ta'rif-2 Mavhum qismining ishoralari bilangina farq qiluvchi kompleks sonlar o'zaro qo'shma kompleks son deyiladi va $\bar{z}$ kabi belgilanadi

Masalan, $z_1 = a_1 + b_1 i$ va $z_2 = a_1 - b_1 i$, z_1 va z_2 qo'shma.

1) $z_1 = 5 + 6i$, $z_1 = 5 - 6i$ 2) $z_1 = -2 - 3i$, $z_1 = -2 + 3i$



Ixtiyoriy kompleks sonni Dekart koordinatalar sistemasida ifodalash mumkin:

Umuman olganda

$i^1 = i$	$i^{4k+1} = i$
$i^2 = -1$	$i^{4k+2} = -1$
$i^3 = -i$	$i^{4k+3} = -i$
$i^4 = 1$	$i^{4k} = 1$

Masalan, $i^5 = i^{4+1} = i$

$i^{15} = i^{3*4+3} = -i$

$N \subset Q \subset R \subset C$

Chizmadan ko'rinadiki, a=r cosφ, b=r sinφ r=$\sqrt{a^2 + b^2}$=$|z|$ – z kompleks sonning moduli,

φ- burchak esa argument deyiladi, $0 \leq \varphi \leq 2\pi k$

Kompleks sonlar ustida arifmetik amallar bajarish.

$z_1 = a_1 + b_1 i$, $z_2 = a_2 + b_2 i$ berilgan bo'lsin.

Ta'rif-3 z_1 va z_2 kompleks sonlarning yig'indisi deb $z_1 + z_2$=$(a_1 + a_2) + (b_1 + b_2)i$ tenglik bilan aniqlanadigan songa aytiladi.

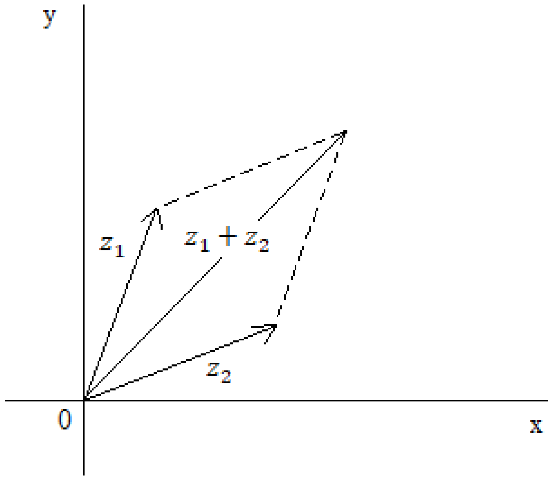
Misol-3 $z_1 = 2 + 5i$, $z_2 = -1 - 3i$

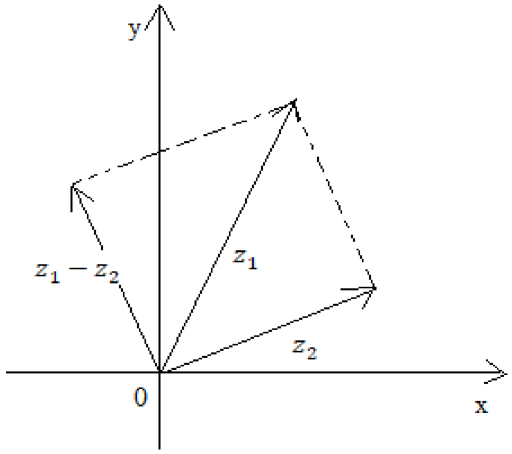
$$z_1 + z_2 = (2 + 5i) + -1 - 3i = (2 - 1) + i(5 - 3) = 1 + 2i$$

Ta'rif-4 z_1 va z_2 kompleks sonlar ayirmasi $z_1 - z_2 = (a_1 - a_2) + (b_1 - b_2)i$ tenglik bilan aniqlanadigan songa aytiladi.

Misol-5 z_1=3-2i, $z_2 = 5 + 4i$

$$z_1 - z_2 = (3 - 5) + (-2 - 4)i = -2 - 6i$$





Ta'rif-5 z_1 va z_2 kompleks sonlar ko'paytmasi deb

$z_1 * z_2 = a_1 * a_2 - b_1 * b_2 + (a_1 * b_2 - a_2 * b_1)i$ tenglik bilan aniqlanadigan songa aytiladi.

Misol-5 $z_1 = 3 + i, \quad z_2 = 2 - 4i$

$$z_1 * z_2 = 3 * 2 - 1 * (-4) + (3 * (-4) - 2 * 1)i = 6 + 4 + (-12 - 2)i = 10 - 14i$$

Ta'rif-6 z_1 va z_2 kompleks sonlarning bo'linmasi deb

$\frac{z_1}{z_2} = \frac{x_1 x_2 + y_1 y_2}{x_2^2 + y_2^2} + i\frac{x_2 y_1 - x_1 y_2}{x_2^2 + y_2^2}$ tenglik bilan aniqlanadigan songa aytiladi.

$$\frac{z_1}{z_2} = \frac{z_1 \bar{z}_2}{z_2 \bar{z}_2} = \frac{(a_1 + b_1 i)(a_2 - b_2 i)}{(a_2 + b_2 i)(a_2 - b_2 i)} = \frac{a_1 a_2 - a_1 b_2 i + a_2 b_1 i + b_1 b_2}{a_2^2 + b_2^2}$$
$$= \frac{a_1 a_2 + b_1 b_2}{a_2^2 + b_2^2} + i\frac{a_2 b_1 - a_1 b_2}{a_2^2 + b_2^2}$$

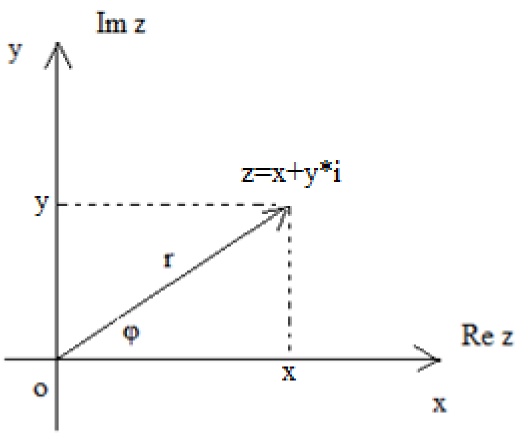
Kompleks sonning trigonometrik shakli

z=x+y*i kompleks sonlarning algebraik shakli deyiladi.

x=r cosφ r – kompleks son moduli

y=r sinφ φ – kompleks son argumenti

r=$\sqrt{x^2+y^2}$



$z = x + yi$ da x va y o'rniga x=r*cosφ va y=r*sinφ ni qo'ysak:

z=r(cosφ+isinφ) kompleks sonlarning trigonometrik shakli

Misol-1 Kompleks sonning moduli 3 ga, argumenti φ=$\frac{\pi}{4}$ da teng bo'lsa, uning haqiqiy va mavhum qismini toping

x=r*cos φ=3*cos $\frac{\pi}{4}$=3*$\frac{\sqrt{2}}{2}$=$\frac{3\sqrt{2}}{2}$

y=r*sin φ=3*sin $\frac{\pi}{4}$=3*$\frac{\sqrt{2}}{2}$=$\frac{3\sqrt{2}}{2}$

Misol-2 z=i kompleks sonning argumentini toping.

x=0 , y=1 , r=$\sqrt{x^2 + y^2} = 1$, x=r*cos φ=0, y=r*sin φ=1 , cos φ=0, sin φ=1, φ=$\frac{\pi}{2}$

Trigonometrik shaklda berilgan kompleks sonlarni ko'paytirish va bo'lish:

$z_1 = r_1(cos\varphi_1 + isin\varphi_1)$; $z_2 = r_2(cos\varphi_2 + isin\varphi_2)$ bo'lsin.

$z_1 z_2 = r_1 r_2(\cos(\varphi_1 + \varphi_2) + isin(\varphi_1 + \varphi_2)$

$\frac{z_1}{z_2} = \frac{r_1}{r_2}(\cos(\varphi_1 - \varphi_2) + isin(\varphi_1 - \varphi_2);$

Kompleks sonni darajaga ko'tarishda Muavr formulasidan foydalaniladi:

$\boldsymbol{z^n = r^n(\cos n\varphi + isinn\varphi)}$; ya'ni

$\boldsymbol{z^n = (r^n\ (cos\varphi + isin\varphi))^n = r^n(\cos n\varphi + isinn\varphi)}$

Misol: $z^3 = [2(cos\frac{\pi}{3} + isin\frac{\pi}{3})]^3$=$2^3(cos3 * \frac{\pi}{3} + isin3 * \frac{\pi}{3})$=8(cosπ+isinπ)=8(-1+i*0)=-8

Kompleks sondan ildiz chiqarish:

Z=r$(cos\varphi + isin\varphi)$ berilgan bo'lsin.

$$\sqrt[n]{\boldsymbol{z}} = \sqrt[n]{\boldsymbol{r}}\left(\boldsymbol{cos}\frac{\boldsymbol{\varphi + 2\pi k}}{\boldsymbol{n}} + \boldsymbol{isin}\frac{\boldsymbol{\varphi + 2\pi k}}{\boldsymbol{n}}\right), k = 0,1,2, \ldots n - 1$$

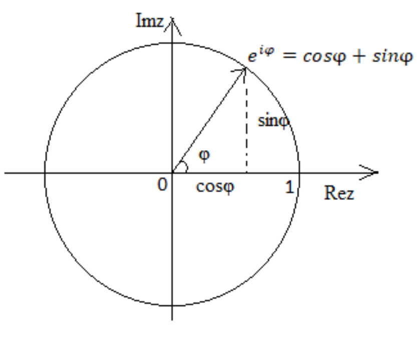
Misol-3 $\sqrt[3]{1 + i}$, avvalo uni trigonometric shaklini topamiz

z=1+i : r=$\sqrt{1 + 1} = \sqrt{2}$ a=r*cosφ=$\sqrt{2}$ cosφ=1 cosφ=$\frac{1}{\sqrt{2}}$ φ=$\frac{\pi}{4}$

Shu tariqa $\sqrt[3]{1 + i} = \sqrt[3]{\sqrt{2}(cos\frac{\pi}{4} + isin\frac{\pi}{4}} = \sqrt[6]{2}(cos\frac{\frac{\pi}{4}+2\pi k}{3} + isin\frac{\frac{\pi}{4}+2\pi k}{3})$, k=0,1,2

k=0 da, k=1 da, k=2 da ildizlar hisoblab qo'yiladi;

Kompleks sonning ko'rsatkichli shaklda yozilishi



z=r*$e^{i\varphi}$, bu yerda

r=$|z|$ – moduli φ-argumenti

$\boldsymbol{e^{i\varphi}}$**=cosφ+isinφ** Eyler formulasi

Misol-4 $z=\frac{3\sqrt{2}}{4}+i\frac{3\sqrt{2}}{4}$ kompleks sonni ko'rsatkichli ko'rinishga o'tkazing.

$a=\frac{3\sqrt{2}}{4}$, $b=\frac{3\sqrt{2}}{4}$, $r=\sqrt{a^2+b^2}=\sqrt{\frac{9*2}{16}+\frac{9*2}{16}}=\sqrt{\frac{9+9}{8}}=\sqrt{\frac{18}{8}}=\frac{3}{2}$;

$a=r*\cos\varphi$ $\quad \frac{3\sqrt{2}}{4}=\frac{3}{2}\cos\varphi$ $\quad \cos\varphi=\frac{3\sqrt{2}}{4}:\frac{3}{2}=\frac{3\sqrt{2}}{4}*\frac{2}{3}=\frac{\sqrt{2}}{2}$

$b=r*\sin\varphi$ $\quad \cos\varphi=\frac{\sqrt{2}}{2}$ $\quad \varphi=\frac{\pi}{4}$

Shunday qilib, $z=\frac{3\sqrt{2}}{4}+i\frac{3\sqrt{2}}{4}=\frac{3}{2}e^{i*\frac{\pi}{4}}$

Ko'rsatkichli shaklda berilgan kompleks sonlarni darajaga ko'tarish va ildiz chiqarish:

$z_1=r_1e^{i\varphi_1}$, $z_2=r_2e^{i\varphi_2}$ berilgan bo'lsin

1) $z_1*z_2=r_1*r_2*e^{i(\varphi_1+\varphi_2)}$
2) $\frac{z_1}{z_2}=\frac{r_1}{r_2}e^{i(\varphi_1-\varphi_2)}$
3) $z^n=r^ne^{i\varphi n}$
4) $\sqrt[n]{z}=\sqrt[n]{r}e^{i\frac{\varphi+2\pi k}{n}}$

Takrorlash uchun savollar:

1) Kompleks son ta'rifi.
2) $z_1=2+3i$, $z_2=-1+i$ bo'lsa, $z_1+z_2, z_1-z_2, z_1*z_2, \frac{z_1}{z_2}$ ni hisoblang
3) $z=\frac{3\sqrt{2}}{2}+\frac{3}{2}i$ ni trigonometric shaklda yozing.
4) $z^4=[\frac{1}{2}(\cos\frac{\pi}{12}+i\sin\frac{\pi}{12})]^4$ ni toping (Muavr formulasi)
5) $\sqrt[5]{z}=\sqrt[5]{6(\cos\frac{\pi}{2}+i\sin\frac{\pi}{2}}=?$
6) $z=\frac{2\sqrt{2}}{3}+\frac{1}{2}i$ ni ko'rstatkichli ko'rinishga o'tkazing.

Foydalanilgan adabiyotlar:

1. Oliy matematika. (Yo.Soatov) 1996-y. Toshkent.
2. Oliy matematika. (F.Rajabov) 2007-y. Toshkent.
3. Курс математического анализа. (Л. Кудрявцев) 1998-г. Москва.
4. www.mathprofi.ru internet sayti.