

Mavzu-13. Funksiyaning nuqtadagi hosilasi. Hosilaning mexanik, geometrik, iqtisodiy, kimyoviy va boshqa talqinlari. Hosila olishning asosiy qoidalari. Murakkab va teskari funktsiyaning hosilalari. Oshkormas va parametrik ko'rinishdagi funksiyalarni differensiallash.

Reja:

1. Hosilaning ta'rifi.
2. Hosilaning mexanik, geometrik, iqtisodiy, kimyoviy talqinlari.
3. Elementar funksiyalar hosilalari.
4. Murakkab va teskari funktsiyaning hosilasi.

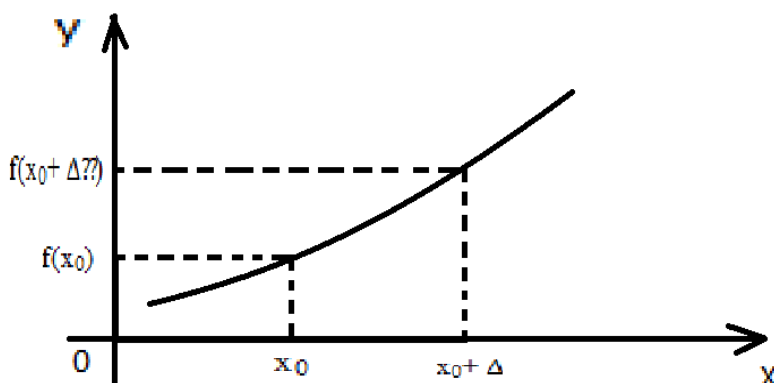
$y=f(x)$ funksiya R da berilgan bo'lsin.

Ta'rif-1 $y=f(x)$ funktsiyaning argumenti orttirmasi deb argumentning x va x_0 nuqtalardagi qiymatlari ayirmasiga aytiladi va Δx kabi belgilanadi:

$$\Delta x = x - x_0$$

Ta'rif-2 $y=f(x)$ funktsiyaning orttirmasi deb funksiyaning x_0 va $x_0 + \Delta x$ nuqtalardagi qiymatlari ayirmasiga aytiladi va Δy yoki Δf kabi belgilanadi:

$$\Delta y = f(x_0 + \Delta x) - f(x_0)$$



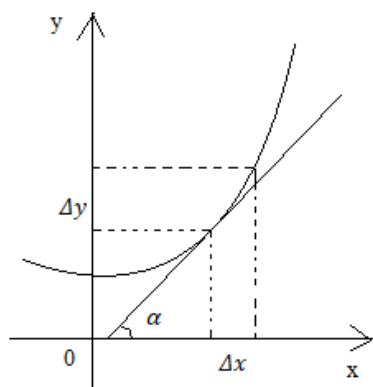
Ta'rif-3 $y=f(x)$ funktsiyaning x_0 nuqtadagi hosilasi deb $y=f(x)$ ning x_0 nuqtadagi funksiya orttirmasi Δy ning argument orttirmasi Δx ga nisbatining argument orttirmasi nolga intilgandagi limitiga aytiladi va y' yoki $f'(x)$ kabi belgilanadi:

$$y' = f'(x) = \lim_{\Delta x \rightarrow 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \rightarrow 0} \frac{f(x_0 + \Delta x) - f(x_0)}{\Delta x}$$

Hosilaning mexanik talqini

$S=S(t)$ funksiya berilgan bo'lsin. S funksiya t vaqt oralig'ida nuqtaning bosib o'tgan masofasi. $v(t)$ nuqtaning tezligi. Hosila bu t vaqtadagi oniy tezlikni bildiradi: $v(t) = S'(t)$

Hosilaning geometrik talqini



$y=f(x)$ funksiyaning x_0 nuqtadagi hosilasi funksiyaning x_0 nuqtadagi urinmasining OX o'qining musbat yo'nalishi bilan tashkil etgan burchagining tangensiga teng:

$$\operatorname{tg} \alpha = \frac{\Delta y}{\Delta x} \quad \left(\begin{array}{l} \Delta y = \sin \alpha \\ \Delta x = \cos \alpha \end{array} \right)$$

Hosilaning iqtisodiy talqini

$y=f(x)$ funksiya biror t vaqt oralig'ida ishlab chiqarilgan mahsulotni bildirsin.

$$y' = \frac{\Delta y}{\Delta x} = \frac{f(x_0 + \Delta x) - f(x_0)}{\Delta t} \quad t\text{-vaqt oralig'ida mehnat unumdorligini bildiradi.}$$

Hosilaning kimyoviy talqini

Hosilaning kimyoviy talqini- bu kimyoviy reaksiyaning tezligini anglatadi. Kimyoviy reaksiyaning tezligi bu vaqt oralig'ida moddalar konsentratsiyasining o'zgarish tezligi.

Hosila olish qoidalari

1. $(u + v)' = u' + v'$
2. $(u - v)' = u' - v'$
3. $(uv)' = u'v + uv'$
4. $\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$

Elementar funksiyalar hosilasi

1. $C' = 0, c = \text{const}$
2. $x' = 1$
3. $(cx)' = c$
4. $(x^n)' = nx^{n-1}$
5. $(\sqrt{x})' = \frac{1}{2\sqrt{x}}$
6. $\left(\frac{1}{x}\right)' = -\frac{1}{x^2}$
7. $(a^x)' = a^x \ln a$
8. $(\ln x)' = \frac{1}{x}$
9. $(\sin x)' = \cos x$
10. $(\cos x)' = -\sin x$

11. $(\operatorname{tg} x)' = \frac{1}{\cos^2 x}$
12. $(\operatorname{ctg} x)' = -\frac{1}{\sin^2 x}$
13. $(\arcsin x)' = \frac{1}{\sqrt{1-x^2}}$
14. $(\arccos x)' = -\frac{1}{\sqrt{1-x^2}}$
15. $(\operatorname{arctg} x)' = \frac{1}{1+x^2}$
16. $(\operatorname{arcctg} x)' = -\frac{1}{1+x^2}$
17. $(\log_a x)' = \frac{1}{x \ln a}$
18. $(e^x)' = e^x$

Murakkab funksiyalarning hosilasi

$y=f(x)$, $x=x(t)$ bo'lsin

Murakkab funksiyaning hosilasi $y' = f'(x)x'(t)$ kabi bo'ladi

Misol 1) $y = (4x+2)^3$

$$y' = 3(4x+2)^2(4x+2)' = 12(4x+2)^2$$

$$2) y = \sqrt{x^2+2}$$

$$y' = \frac{1}{2\sqrt{x^2+2}}(x^2+2)' = \frac{x}{\sqrt{x^2+2}}$$

Teskari funksiyaning hosilasi

$y=f(x)$, $x=g(y)$ teskari funksiyalar bo'lsin

$$y' = \frac{1}{g'(y)}$$

Misol $y=\arcsin x$, $x=\sin y$

$$y' = \frac{1}{(\sin y)'} = \frac{1}{\cos y} = \frac{1}{\sqrt{1-\sin^2 x}} = \frac{1}{\sqrt{1-x^2}}$$

Takrorlash uchun savollar:

Quyidagi funksiyalarning hosilalarini toping.

1. $y = 3x^4 + 2x^3 + x^2 - 6$
2. $y = (5x+6)^2 - 3$
3. $y = \sin x \cos 3x$
4. $y = \sqrt{5x^2 - 2}$
5. $y = \arcsin(5x+1)$
6. $y = e^{5x}$

Foydalanilgan adabiyotlar:

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2. Matematika. (A. Sanginov) 2011-y. Toshkent.
3. Курс математического анализа. (Л. Кудрявцев) 1998-г. Москва.
4. www.mathprofi.ru internet sayti.